Electromagnetic tomography (EMT): image reconstruction based on the inverse problem*

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Abstract Starting from Maxwell's equations for inhomogeneous media, nonlinear integral equations of the inverse problem of the electromagnetic tomography (EMT) are derived, whose kernel is the dyadic Green's function for the EMT sensor with a homogeneous medium in the object space. Then in terms of ill-posedness of the inverse problem, a Tikhonov-type regularization model is established based on a linearization-approximation of the nonlinear inverse problem. Finally, an iterative algorithm of image reconstruction based on the inverse problem and reconstruction images of some object flows for simplified sensor are given. Initial results of the image reconstruction show that the algorithm based on the inverse problem is superior to those based on the linear back-projection in the quality of image reconstruction.

Keywords: **electromagnetic theory, inverse problem, tomography, image reconstruction.**

Process tomography is a technique to process the data collected from an array of sensors in order to obtain quantitative information about distribution of multi-component flows at an inaccessible location in industrial process plants. In the 1990s Yu et al. first proposed a tomography system with a parallel field sensor based on electromagnetic induction (EMT)[1]. Then Xiong et al. studied theoretical basis of the EMT forward problem and dyadic Green's function method formulating the EMT inverse problem[2~4]. This system consists of three main subsystems: a primary sensor, a control and data processing circuitry and a computer for image reconstruction. The basic mechanism of this system is that the excitation coils acquire the modulated signal from the control circuit and generate an excitation field (for example, a parallel field) in the object space. The field signals distorted by the measured materials within the object space are detected by the detection soils and then are fed into the computer via the data processing circuit to create a reconstruction image of the object space on the screen with the aid of an image reconstruction software. The EMT is an invasive or non-contacting tomography technique, and it can acquire information on the distribution of conductivity and permeability in the object space simultaneously, therefore it has potential in applications of many industrial processes such as oil/sea water transportation in the oil industry, air/molten metal flow in the steel industry, water/metallic ores in the mineral industry, cracks of rotating parts in engineering industry, foreign metal matter in the food industry and so on.

The image reconstruction is the crux of EMT. This paper aims at deriving the mathematical expression of the EMT inverse problem with the electromagnetic theory for inhomogeneous medium and giving the method of the linearization and regularization of the non-linearity and ill-posedness of the inverse problem in order to develop an iterative algorithm based on the inverse problem.

1 Mathematical formulations of the inverse problem

For an EMT system two problems have to be solved, namely the forward and inverse problems. The forward problem is to determine the values measured by detectors for a distribution of conductivity and permeability in the object space. For the forward problem simulation is an important means besides measurement. The inverse problem is to determine the distribution of the conductivity and permeability of the material within the object space from the detected signals, that is, image reconstruction.

The structure of the EMT sensor is shown in

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Fig. 1. Ω denotes the space occupied by the sensor; s_0 is the location where the detection coils lie, Γ and $\boldsymbol{n}$ are the inner boundary and inward normal unit vector of the conducting screen, respective-ly; $\boldsymbol{r}$ is a location vector of field point, $\varepsilon_b(\boldsymbol{r})$, $\sigma_b(\boldsymbol{r})$, $\mu_b(\boldsymbol{r})$ the distributions of permittivity, conductivity and permeability of the material in Ω, as there is only background medium within the object space D_0. And $\varepsilon(\boldsymbol{r})$, $\sigma(\boldsymbol{r})$, $\mu(\boldsymbol{r})$ are correspondent distributions when there is also inhomogeneous objects within D_0.

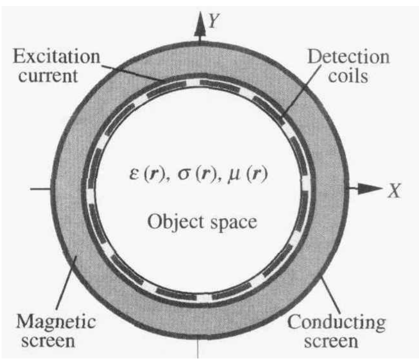

Fig. 1. A cross-section of the sensor.

According to the electromagnetic theory the boundary value problem for electric field $\boldsymbol{E}$ can be determined by equations

$$\nabla\times\mu^{-1}(\boldsymbol{r})\nabla\times\boldsymbol{E}(\boldsymbol{r})-\omega^2\varepsilon^*(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r})=\mathrm{j}\omega\boldsymbol{J}(\boldsymbol{r}),\quad \boldsymbol{r}\in\Omega,$$
$$\boldsymbol{n}\times\boldsymbol{E}(\boldsymbol{r})=0,\quad \boldsymbol{r}\in\Gamma, \tag{1}$$

where $\varepsilon^*(\boldsymbol{r})=\varepsilon(\boldsymbol{r})-\mathrm{j}\dfrac{\sigma(\boldsymbol{r})}{\omega}$ denotes the equivalent permittivity of conductive medium, $\boldsymbol{J}(\boldsymbol{r})$ is the density of the exciting current. Introducing the electric dyadic Green's function of the first kind for the EMT sensor $\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')$, $\boldsymbol{r}'$ denotes the location vector of a source point. The integral equation for the electric field can be derived as

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r})=&\,\mathrm{j}\omega\int_\Omega \mathrm{d}\boldsymbol{r}'\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\mu_b(\boldsymbol{r}')\boldsymbol{J}(\boldsymbol{r}')\\&+\omega^2\int_\Omega \mathrm{d}\boldsymbol{r}'\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\mu_b(\boldsymbol{r}')(\varepsilon^*(\boldsymbol{r}')\\&-\varepsilon_b(\boldsymbol{r}'))\boldsymbol{E}(\boldsymbol{r}')\\&-\int_\Omega \mathrm{d}\boldsymbol{r}'\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\mu_b(\boldsymbol{r}')\nabla'\\&\times(\mu^{-1}(\boldsymbol{r}')-\mu_b^{-1}(\boldsymbol{r}'))\nabla'\times\boldsymbol{E}(\boldsymbol{r}'),\\&\boldsymbol{r}\in\Omega.\end{aligned}\tag{2}$$

An expression of the electric dyadic Green's function $\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')$ for EMT sensor of a layered cylinder can be deduced by addition of the reflected wave terms caused by the layered media on the basis of the eigenfunction expansions of the dyadic Green's function for a single hollow conductive cylinder[4]. The physical meanings of three terms on the right-hand side of (2) are: the first term represents the electric excitation field $\boldsymbol{E}_{\mathrm{exc}}(\boldsymbol{r})$, the second integral characterizes the electric field produced by the polarization current and/or conduction current induced within the materials. The third integral corresponds to the electric field produced by the magnetization source induced within the materials. The electric field denoted by the second and third terms is designated as the object field $\boldsymbol{E}_{\mathrm{obj}}(\boldsymbol{r})$. Assuming that displacement current is ignored, from (2) we have

$$\begin{aligned}\boldsymbol{E}_{\mathrm{obj}}(\boldsymbol{r})=&-\mathrm{j}\omega\int_\Omega \mathrm{d}\boldsymbol{r}'\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\mu_b(\boldsymbol{r}')\sigma(\boldsymbol{r}')\boldsymbol{E}(\boldsymbol{r}')\\&-\mathrm{j}\omega\int_\Omega \mathrm{d}\boldsymbol{r}'\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\mu_b(\boldsymbol{r}')\nabla'\\&\times(\mu^{-1}(\boldsymbol{r}')-\mu_b^{-1}(\boldsymbol{r}'))\mu(\boldsymbol{r}')\boldsymbol{H}(\boldsymbol{r}'),\\&\boldsymbol{r}\in\Omega.\end{aligned}\tag{3}$$

The first term in (3) is related to $\sigma(\boldsymbol{r}')$ and the second term depends on $\mu(\boldsymbol{r}')$, that is, (3) implies a dual modality imaging. For a special case, both the background medium (e.g. air) and the detected materials (e.g. copper) are non-magnetic ($\mu=\mu_b=\mu_0$), the third term in (3) equals zero, so we have

$$\boldsymbol{E}_{\mathrm{obj}}(\boldsymbol{r})=-\mathrm{j}\omega\int_\Omega \mathrm{d}\boldsymbol{r}'\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}')\cdot\mu_b(\boldsymbol{r}')\sigma(\boldsymbol{r}')\boldsymbol{E}(\boldsymbol{r}'),\quad \boldsymbol{r}\in\Omega. \tag{4}$$

As $\boldsymbol{E}(\boldsymbol{r}')$ itself is a functional of $\sigma(\boldsymbol{r}')$, $\boldsymbol{E}_{\mathrm{obj}}(\boldsymbol{r})$ is a nonlinear functional of $\sigma(\boldsymbol{r}')$. The problem to find $\sigma(\boldsymbol{r}')$ from given $\boldsymbol{E}_{\mathrm{obj}}(\boldsymbol{r})$ belongs to a non-linear inverse electromagnetic problem that is to solve the Fredholm integral equation of the first kind.

2 Tikhonov-type regularization of the inverse problem

The nonlinear inverse problem formulated in Section 1 is ill-posed in the sense of Hadamard[5], that is, the conditions of existence, uniqueness and stability of the solution are not satisfied. It is necessary that the inverse problem is regularized in order to obtain an approximate solution that is meaningful physically and is stable numerically. For simplicity, and without loss of generality, the image reconstruction of the copper/air flow formulated in (4) is considered and the effect of the layered structure of the sensor on the parallel excitation field can be ignored, that corresponds to the cylindrical object space lies in the free space. In this case the dyadic Green's function for a two-dimensional transverse magnetic (TM) wave can be taken as the dyadic Green's function in the integral Eq. (4):

$$G(\boldsymbol{r}, \boldsymbol{r}') = \frac{\mathrm{j}}{4}\boldsymbol{H}_0^{(1)}(k_0 \mid \boldsymbol{r} - \boldsymbol{r}' \mid), \quad (5)$$

where $\boldsymbol{H}_0^{(1)}$ denotes the Hankel function of the first kind of the zero order and $k_0 = \omega^2\mu_0\varepsilon^* \approx -\mathrm{j}\omega\mu_0\sigma$ (when the displacement current is ignored). The integral Eqs. (2) and (4) can be written as (the suffix p denotes the pth projection):

$$\boldsymbol{E}_\mathrm{p}(\boldsymbol{r}) = \boldsymbol{E}_{\mathrm{p,exc}}(\boldsymbol{r}) + \int_{D_0} \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{r}')Z(\boldsymbol{r}')\boldsymbol{E}_\mathrm{p}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}', \quad \boldsymbol{r} \in D_0, \quad (6)$$

$$\dot{E}_{\mathrm{p,obj}}(\boldsymbol{r}) = \int_{D_0} \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{r}')Z(\boldsymbol{r}')\dot{E}_\mathrm{p}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}', \quad \boldsymbol{r} \in s_0, \quad (7)$$

where

$$Z(\boldsymbol{r}') = -\mathrm{j}\omega\mu_0\sigma(\boldsymbol{r}'), \quad \boldsymbol{r}' \in D_0, \quad (8)$$

which corresponds to the distribution of the unknown conductivity.

The object space D_0 is divided into K elementary units and the values of the object field are measured at M points. The use of the moment method, by choosing pulse function basis and point matching, can transform integral Eqs. (6) and (7) into the matrix relations respectively:

$$[\boldsymbol{E}_{\mathrm{p,exc}}] = [\boldsymbol{I} - \boldsymbol{G}_\mathrm{K}\boldsymbol{Z}][\boldsymbol{E}_\mathrm{p}], \quad (9)$$

$$[\boldsymbol{E}_{\mathrm{p,obj}}] = [\boldsymbol{G}_\mathrm{M}][\boldsymbol{Z}][\boldsymbol{E}_\mathrm{p}], \quad (10)$$

where $[\boldsymbol{E}_{\mathrm{p,exc}}]$ and $[\boldsymbol{E}_\mathrm{p}]$ are K vectors, $[\boldsymbol{I}]$ is a $K \times K$ identity matrix, $[\boldsymbol{G}_\mathrm{K}]$ the $K \times K$ symmetric matrix, $[\boldsymbol{Z}]$ the diagonal $K \times K$ matrix, $[\boldsymbol{G}_\mathrm{M}]$ the $M \times K$ matrix, $[\boldsymbol{E}_{\mathrm{p,obj}}]$ the M vector.

Considering small variations of the conductivity, (9) and (10) become respectively

$$[\Delta\boldsymbol{E}_\mathrm{p}] = [\boldsymbol{G}_\mathrm{K}][\Delta(\boldsymbol{ZE}_\mathrm{p})], \quad (11)$$

$$[\Delta\boldsymbol{E}_{\mathrm{p,obj}}] = [\boldsymbol{G}_\mathrm{M}][\Delta(\boldsymbol{ZE}_\mathrm{p})]. \quad (12)$$

The quantity $[\Delta(\boldsymbol{ZE}_\mathrm{p})]$ can be approximated linearly:

$$[\Delta(\boldsymbol{ZE}_\mathrm{p})] \approx [\Delta\boldsymbol{Z}][\boldsymbol{E}_\mathrm{p}] + [\boldsymbol{Z}][\Delta\boldsymbol{E}_\mathrm{p}]. \quad (13)$$

Introducing (11) into (13) gives

$$[\Delta(\boldsymbol{ZE}_\mathrm{p})] = [\boldsymbol{I} - \boldsymbol{ZG}_\mathrm{K}]^{-1}[\Delta\boldsymbol{Z}][\boldsymbol{E}_\mathrm{p}]. \quad (14)$$

Noting that $[\boldsymbol{G}_\mathrm{K}]$ is a symmetric matrix, we have

$$[\boldsymbol{I} - \boldsymbol{ZG}_\mathrm{K}]^{-1} = \lfloor[\boldsymbol{I} - \boldsymbol{ZG}_\mathrm{K}]^{-1}\rfloor^*,$$

where the asterisk " $*$ " denotes the conjugate transpose.

Substituting (14) in (12) yields

$$[\Delta\boldsymbol{E}_{\mathrm{p,obj}}] = [\boldsymbol{D}_\mathrm{p}][\Delta\boldsymbol{z}], \quad (15)$$

where $[\Delta\boldsymbol{z}]$ is a K vector whose components are the elements of $[\boldsymbol{Z}]$; $[\boldsymbol{D}_\mathrm{p}]$ is a $M \times K$ matrix that is mathematically called the Frechet derivative, its expression is

$$[\boldsymbol{D}_\mathrm{p}] = [\boldsymbol{G}_\mathrm{M}][\boldsymbol{I} - \boldsymbol{ZG}_\mathrm{K}]^{-1}[\bar{\boldsymbol{E}}_\mathrm{p}], \quad (16)$$

where $[\bar{\boldsymbol{E}}_\mathrm{p}]$ is a diagonal $K \times K$ matrix whose elements are the components of vector $[\boldsymbol{E}_\mathrm{p}]$.

Collecting the quantity $\Delta\boldsymbol{E}_{\mathrm{p,obj}}$ for each view $p = 1, 2, \cdots, P$ leads to the following linear relation:

$$[\Delta\boldsymbol{E}_{\mathrm{obj}}] = [\boldsymbol{D}][\Delta\boldsymbol{z}], \quad (17)$$

where $[\Delta\boldsymbol{E}_{\mathrm{obj}}]$ is a $P \cdot M$ vector, $[\boldsymbol{D}]$ the $P \cdot M \times K$ matrix.

If we let

$$\Delta\boldsymbol{E}_{\mathrm{obj}} = \Delta\boldsymbol{E}_{\mathrm{obj}}^{(k)} = \boldsymbol{E}_{\mathrm{obj}}^{*} - \boldsymbol{E}_{\mathrm{obj}}^{(k)},$$

$$\boldsymbol{D} = \nabla_z^{\mathrm{T}}\boldsymbol{E}_{\mathrm{obj}}^{(k)},$$

$$\Delta\boldsymbol{z} = \Delta\boldsymbol{z}^{(k)} = \boldsymbol{z}^{(k+1)} - \boldsymbol{z}^{(k)},$$

$k = 1, 2, \cdots$, where $\boldsymbol{E}_{\mathrm{obj}}^{*}$ is a measured value of the object field, $\boldsymbol{E}_{\mathrm{obj}}^{(k)}$ the calculated value of the object field at the kth iterated point $\boldsymbol{z}^{(k)}$, $\nabla_z^{\mathrm{T}}\boldsymbol{E}_{\mathrm{obj}}^{(k)}$ the gradient matrix of $\boldsymbol{E}_{\mathrm{obj}}$ with respect to $\boldsymbol{z}$ at iterated point $\boldsymbol{z}^{(k)}$, then solving Eq. (17) is equivalent to solving the inverse problem with the Newton iterative algorithm.

To overcome the ill-posedness of the inverse problem, according to Tikhonov's regularization theory[5], the inverse problem is transformed into the extremum problem of the following functional:

$$J_\alpha(\Delta\boldsymbol{z}^{(k)}) = \| \Delta\boldsymbol{E}_{\mathrm{obj}}^{(k)} - \boldsymbol{D}\Delta\boldsymbol{z}^{(k)} \|^2 + \alpha \| \Delta\boldsymbol{z}^{(k)} \|^2, \quad (18)$$

where α is called a regularization parameter, a choice for α would be made between accuracy and stability (the simulation in this study takes $\alpha = 0.0008$). The optimal perturbation $\Delta\boldsymbol{z}^{(k)}$ makes the extreme value of the above functional equivalent to the solution of the following equation:

$$\Delta\boldsymbol{z}^{(k)} = [\boldsymbol{D}^*\boldsymbol{D} + \alpha\boldsymbol{I}]^{-1}\boldsymbol{D}^*\Delta\boldsymbol{E}_{\mathrm{obj}}^{(k)}, \quad k = 1, 2, \cdots. \quad (19)$$

3 Image reconstruction algorithm and initial results

The image reconstruction algorithm is a numerical iterative method, which includes a few steps as follows.

(i) Given $\boldsymbol{z}^{(k)}$ and solving the forward problem (9) to obtain the total electric field $\boldsymbol{E}^{(k)}$ inside D_0;

(ii) the estimated vector of the object field at the detectors $\boldsymbol{E}_{\text{obj}}^{(k)}$ is given from Eq. (10);

(iii) calculating the error $\Delta \boldsymbol{E}_{\text{obj}}^{(k)} = \boldsymbol{E}_{\text{obj}}^{*} - \boldsymbol{E}_{\text{obj}}^{(k)}$ between the estimated vector and the measured vector;

(iv) the perturbation $\Delta \boldsymbol{z}^{(k)}$ and updated vector $\boldsymbol{z}^{(k+1)} = \boldsymbol{z}^{(k)} + \Delta \boldsymbol{z}^{(k)}$ are given from relation (19).

The procedures are repeated till an acceptable error $\Delta \boldsymbol{E}_{\text{obj}}^{(k)}$ is achieved. The reconstruction images of various flow-types for 30 iterations are shown in Fig. 2 (ordinate: σ (S/m) and abscissas: $X - Y$ (m)). For the sake of contrast, the image reconstructed by a linear back-projection algorithm[6,7] for a four-object flow-type is also shown in Fig. 2.

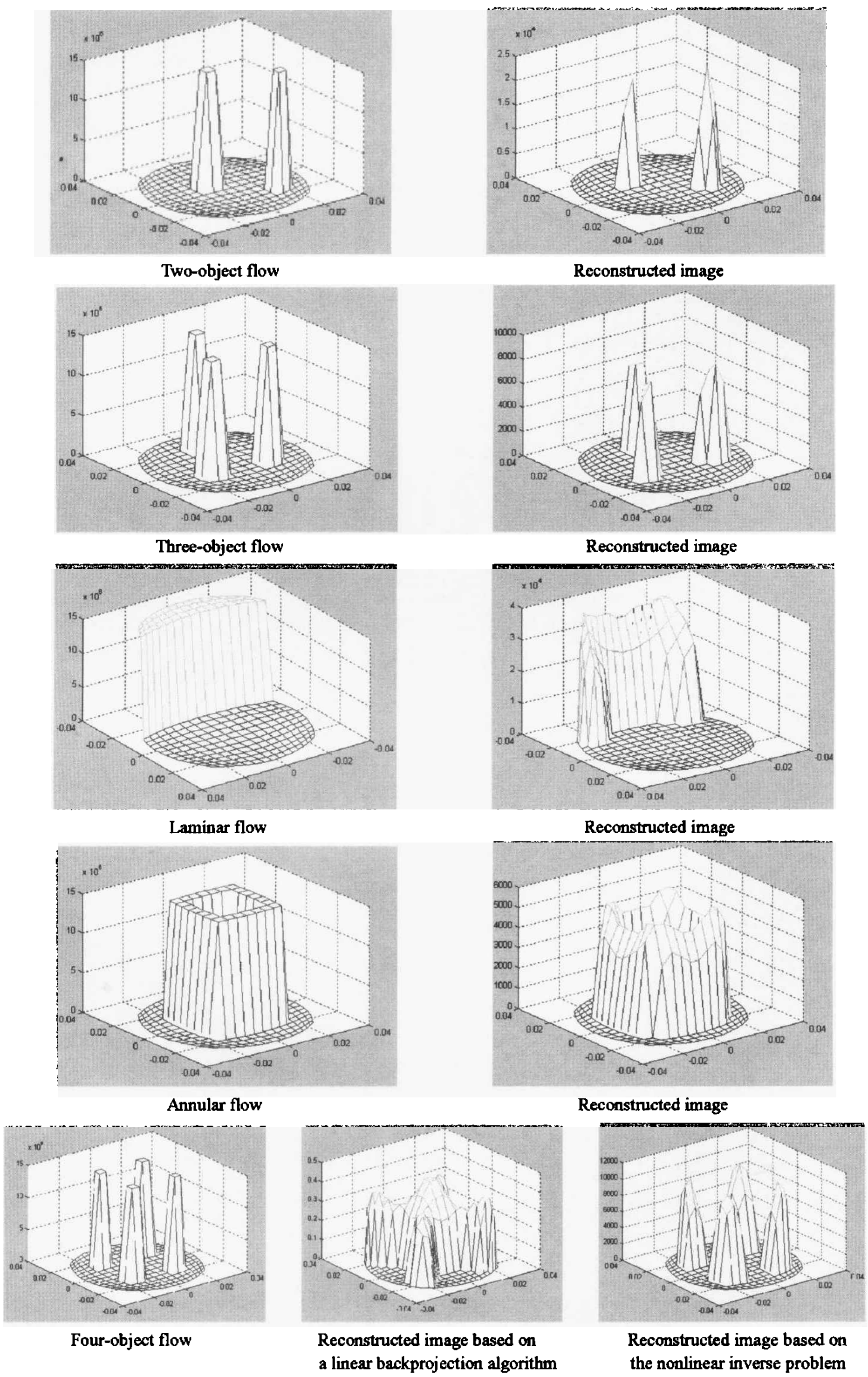

Fig. 2. Reconstructed images based on the regularization of the inverse problem.

4 Conclusions

The dual modality electromagnetic tomography technique can theoretically reconstruct the distribution of conductivity/permeability simultaneously. Because of the properties of the soft-field, complex-field, nonlinear-field and three-dimensional field of EMT, the linear backprojection algorithms can only reconstruct qualitative images of simple object-flows. Quantitative image reconstructions of complex object-flows depend on the solution of the inverse electromagnetic problem of the EMT sensor formulated by the Fredholm integral equation of the first kind.

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